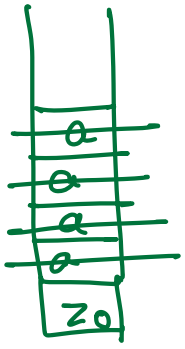
2 Stack PDA

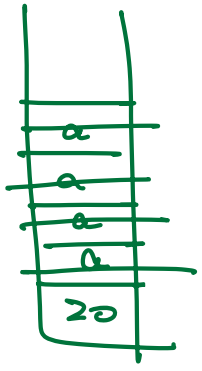
$L = a^n b^n c^n \mid n \geq 1$

Single stack ?

aabbcc ✓✓✓✓✓✓

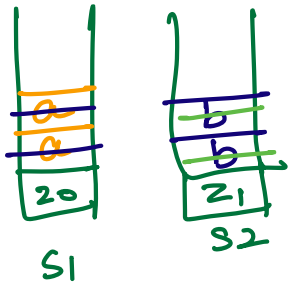


aabccc ✓✓✓✓✓✓



2 Stacks

aabbcc ✓✓✓✓✓✓

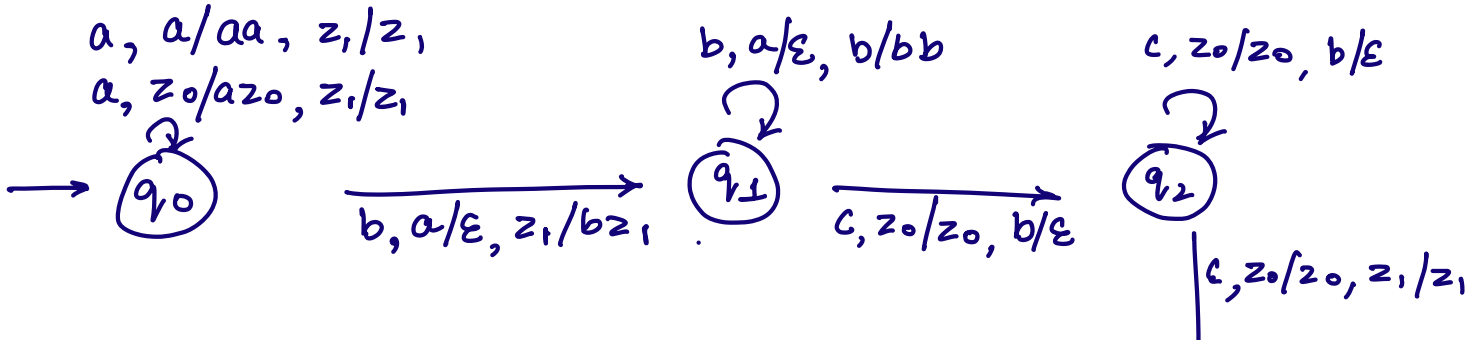


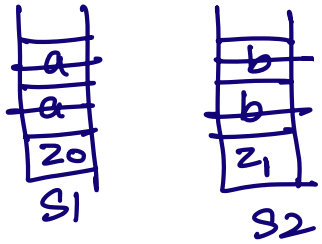
input alphabet Σ = input string terminate

$Q \times \underline{\Sigma \cup \{\varepsilon\}} \times \Gamma \longrightarrow Q \times \Gamma^*$

$Q \times \underline{\Sigma \cup \{\varepsilon\}} \times \Gamma \times \Gamma \longrightarrow Q \times \Gamma^* \times \Gamma^*$

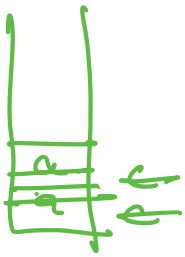
input string : aabbccε ✓✓✓✓✓✓✓



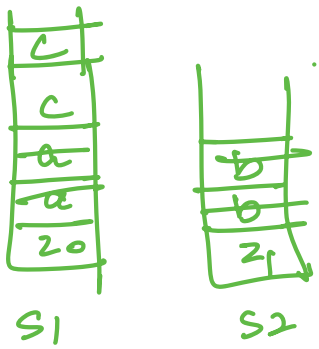


Eg. $L = a^n b^n c^n d^n \mid n \geq 1$

Single stack:



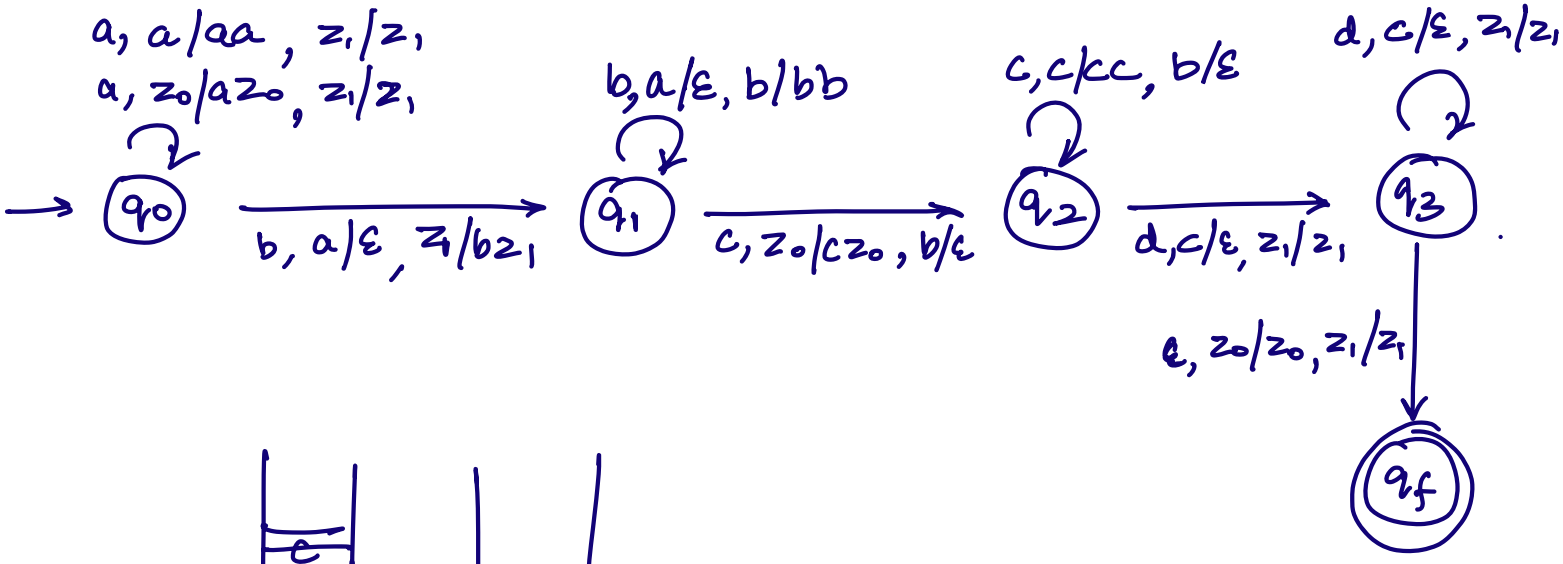
aabbccdd

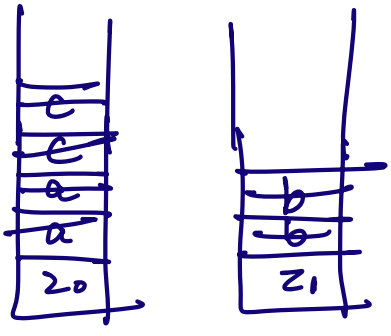


á b ć ć d d

á a b b ć ć d d

á á b b ć ć d d ε





S_1 S_2

CFL ⟶ PDA using 1 stack

Properties of CFL:

CFL closed under union, concatenation & Kleene closure

(2) + (3) ⟶ (5)

NN NN NN

NN are closed under addition

CFL ∪ CFL ⟶ CFL

CFL are closed under union.

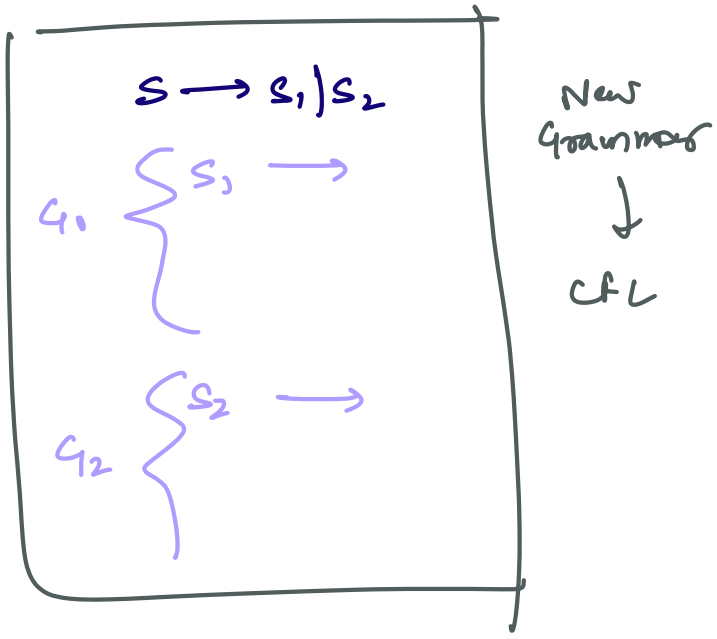
Union:

L_1 ∨ L_2

CFL CFL

G_1 G_2

S_1 S_2



Concatenation:

L_1 L_2

$a^n b^n$ $c^m d^m$

$L_1 \cdot L_2 = a^n b^n c^m d^m$

↳ CFL

S_1 S_2

$S \rightarrow S_1 \cdot S_2$ ✓

L_1 { $S_1 \rightarrow$

L_2 { $S_2 \rightarrow$

Closure:

$L_1 \rightarrow CFL$

$L_1^* \rightarrow CFL$

L_1 { $S_1 \rightarrow$

$S \rightarrow S_1 S \mid \varepsilon$

CFL are not closed under intersection & complementation

Intersection:

L_1 L_2

CFL CFL

$L_1 \cap L_2$ CFL?

$L_1 = \{a^n b^n c^m \mid n, m \geq 0\}$ → CFL

$L_2 = \{a^m b^n c^n \mid n, m \geq 0\}$ →

$L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 0\}$

↳ not a CFL

CFL are not closed under intersection.

Complementation:

$$\overline{L_1 \cap L_2} = \overline{L_1} \cup \overline{L_2}$$

$L_1 \to CFL$

$\overline{L_1} \to CFL \to ??$

Assume: CFL are closed under complementation.

$L_1 \to CFL$ $\quad$ $L_2 \to CFL$

$\overline{L_1} \to CFL$ $\quad$ $\overline{L_2} \to CFL$ $\quad$ (Assume)

$\overline{L_1} \cup \overline{L_2} \longrightarrow CFL$ $\quad$ (Already Proved)

↓ CFL $\quad$ ↘ CFL

$\overline{\overline{L_1} \cup \overline{L_2}} \longrightarrow CFL$ $\quad$ (Assume)

$L_1 \cap L_2 \longrightarrow CFL$

↳ we already know CFL ✗

↳ assumption wrong

CFL are not closed under complementation.

Decidability Problem of CFL:

membership:

String w $\quad$ Language L

$w \in L$?

↓

belong

CFG → CNF form → $NT \to NT.NT$, $NT \to T$, $S \to \varepsilon$ } 2n-1 steps string reach

2n-1 loop

Decidable

w reach x not in the language.

Emptiness:

$L = \phi$

Algo:

→ CFG, Simplify

↳ ε productions remove

↳ unit productions remove

↳ useless symbol remove

→ Start symbol is useless ?

True: Empty

false: Not Empty

Decidable

finiteness:

CFG → Simplify → ε ✗, useless ✗, unit ✗

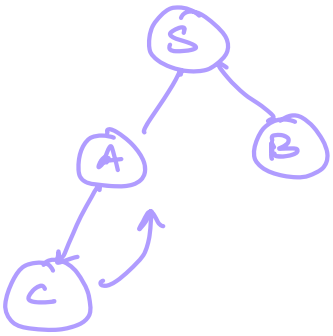
Dependency Graph

$S \to AB$

$A \to aC \mid a$

C → aA | b

B → a



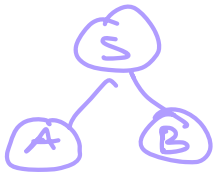
infinite

Dependency Graph, if there is a cycle ?

Yes → Infinite

No → Finite

S → AB
A → a
B → b



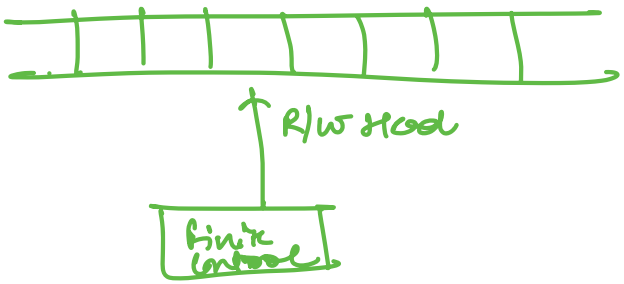
finite

Turing Machine:

- PDA moves only in 1 dir^{n}.

  TM can move in both dir^{n}s.

-



tape
(input string will be present on tape)

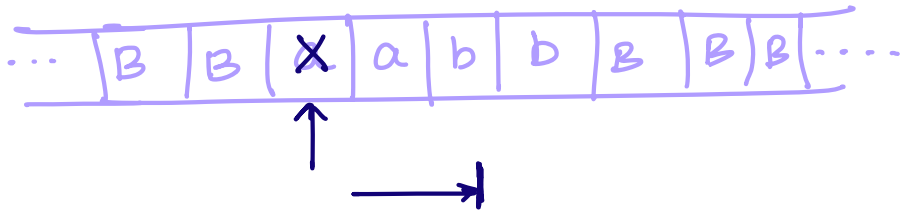
You can read the symbol from the tape & write the symbol on tape

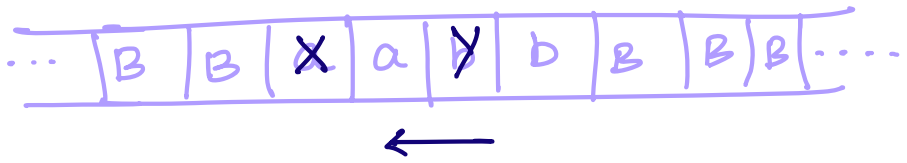
- TM can't accept ε.

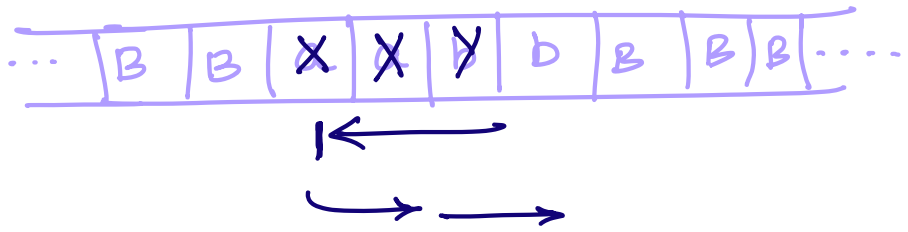
→ 

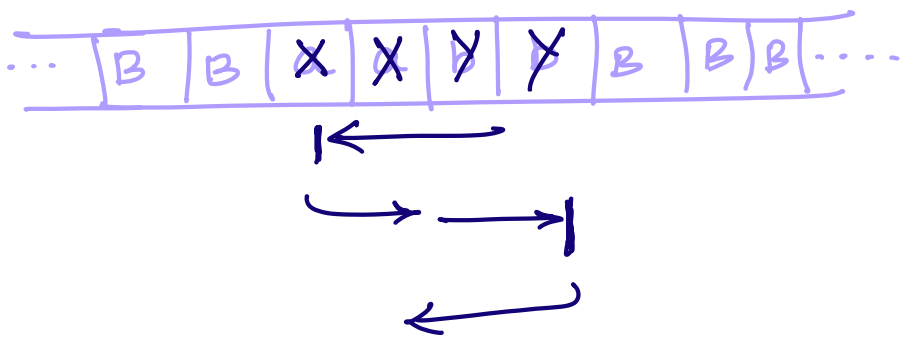
{ε, aa, aaaa . . . }

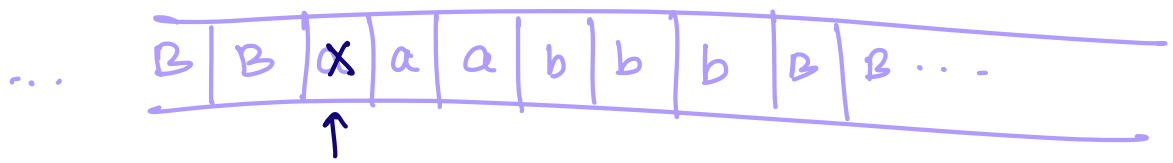
Eg: $a^n b^n \mid n \geq 1$

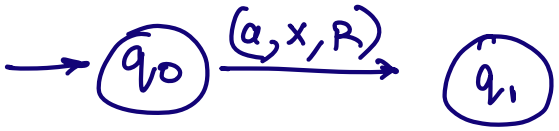


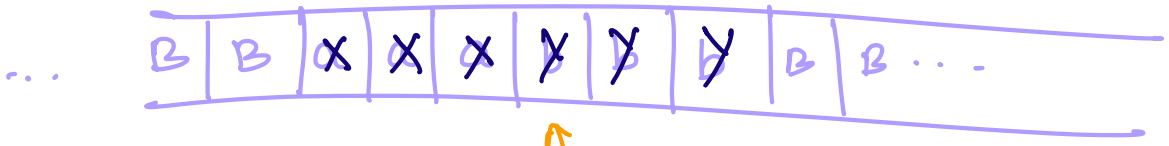


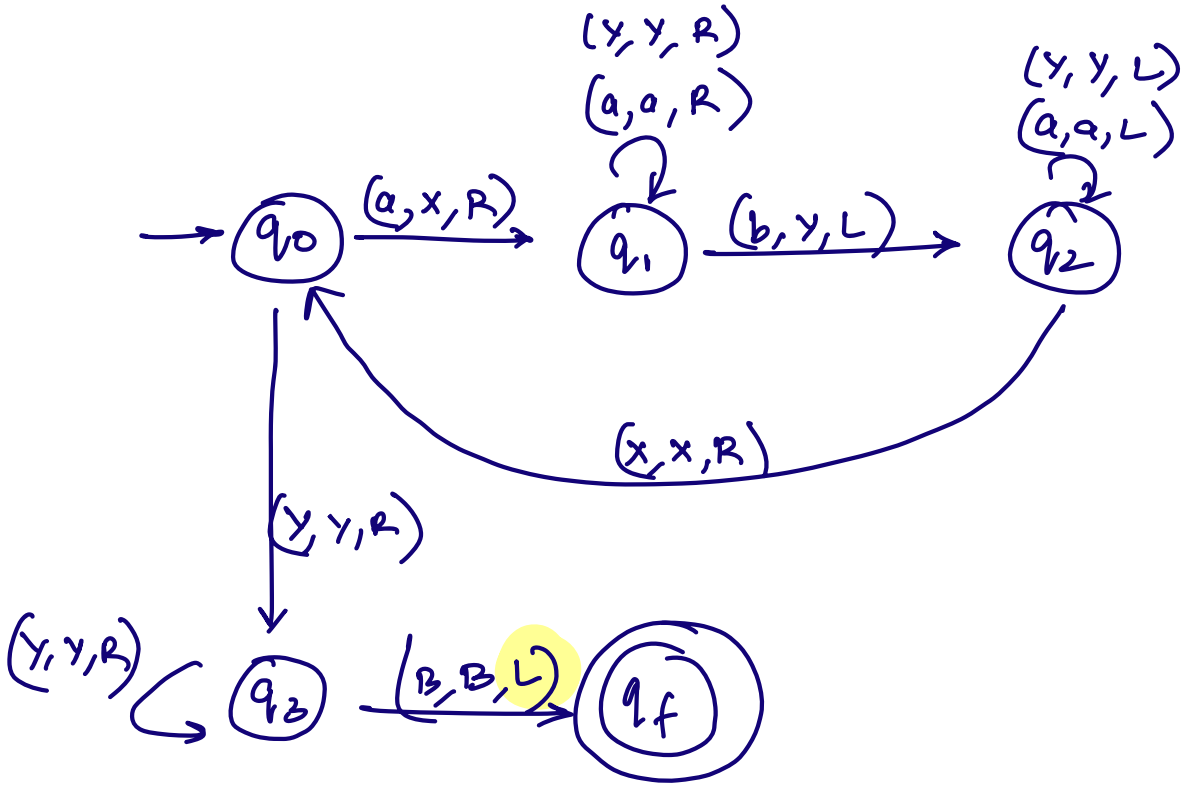




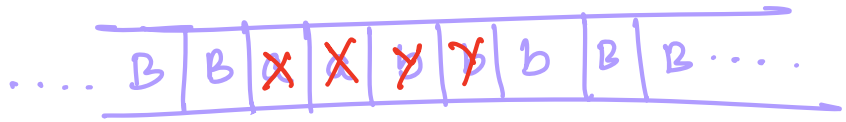






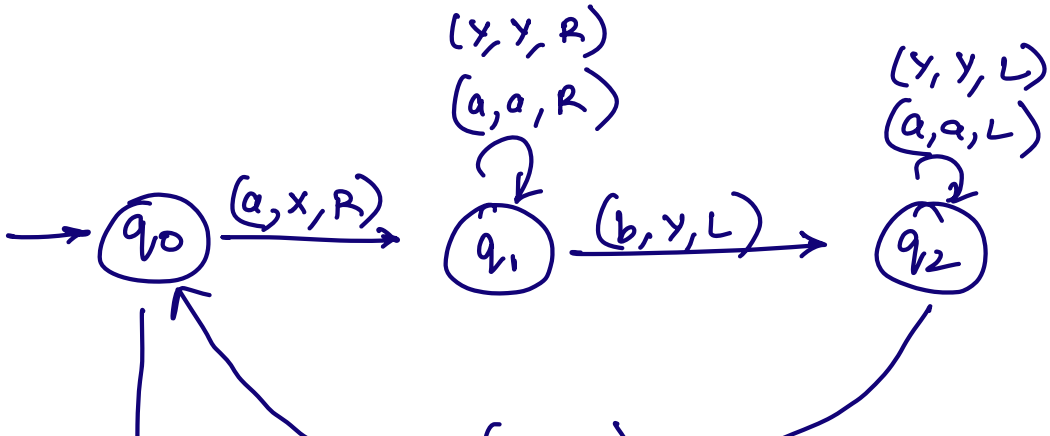


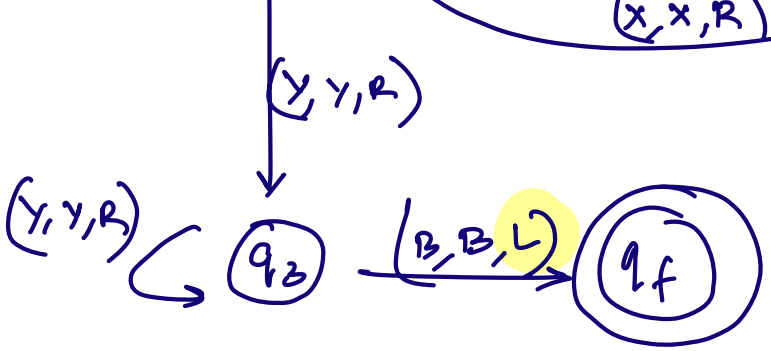
String : aabbb



$q_0 \to q_1 \to q_2 \to q_0 \to q_1 \to q_2 \to q_0 \to q_3$

State: q_3
$\Sigma = b$
↓
no transition





q_1 stuck?
B ?

Stuck

aaabb $q_0 \to q_1 \to q_2 \to q_0 \to q_1 \to q_2 \to q_0 \to q_1$

not accepted.